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Integration of Rational Function

Introduction A function of the form

$\frac{F(x)}{\varphi(x)}$ is called a rational algebraic function if $F(x)$ and $\varphi(x)$ are polynomial functions in x. Thus

$$\frac{x}{a^2+x^2}, \quad \frac{2x^2-3x+1}{x^2+x+1}, \quad \frac{1}{a^2-x^2}$$

are all rational functions.

Additional Standard forms for integration of Rational functions.

1.) $\int \frac{1}{x^2+a^2}$ when integrand ~~is~~ in ~~the~~ form of $\frac{1}{x^2+a^2}$

then Put $x = a\tan\theta$

$\Rightarrow dx = a\sec^2\theta\, d\theta$.

$$I = \int \frac{a\sec^2\theta\, d\theta}{a^2\tan^2\theta + a^2} = \int \frac{a\sec^2\theta\, d\theta}{a^2(\tan^2\theta+1)}$$

$$= \frac{1}{a}\int \frac{\sec^2\theta}{\sec^2\theta}\, d\theta = \frac{1}{a}\theta + C$$

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$\frac{1}{a} \tan^{-1}\frac{x}{a} + C$

ii.) $\int \frac{1}{x^2-a^2}\, dx \quad (x > a)$

When integrand is in the form

$\frac{1}{x^2-a^2}$ Putting ~~xe~~ $\frac{1}{x^2-a^2} = \frac{1}{(x-a)(x+a)}$

$= \frac{1}{2a} \int \frac{(x+a)-(x-a)}{(x-a)(x+a)}\, dx$

$= \frac{1}{2a}\left[\int \left(\frac{1}{x-a} - \frac{1}{x+a}\right) dx\right.$

$= \frac{1}{2a}\left[\log(x-a) - \log(x+a)\right]$

$= \frac{1}{2a} \log\frac{(x-a)}{x+a}, \quad x > a$

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iii) $\int \frac{1}{a^2-x^2}\,dx \quad (x<a)$

$$I = \int \frac{1}{a^2-x^2}\,dx$$

$$= \int \frac{1}{(a+x)(a-x)}\,dx.$$

$$= \frac{1}{2a}\int \frac{(a-x)-(a+x)}{(a+x)\cdot(a-x)}\,dx$$

$$= \frac{1}{2a}\int \left\{\frac{1}{a+x} - \frac{1}{(a-x)}\right\} dx$$

$$= \frac{1}{2a}\left(\log(a+x) - \log(a-x)\right)$$

$$= \frac{1}{2a}\log\frac{a+x}{a-x} + c \quad x<a$$